Sun's Effect on Planetary Temperatures

The total power (energy per second), P, radiated by objects such as the Sun per square meter is given by the Stefan-Boltzmann law

$$P = \sigma T^4$$

where T is the temperature, and σ is the Stefan-Boltzmann constant and is equal to 5.67×10^{-8} W m^{-2} K^{-4}.

The temperature of the Earth can now be estimated based on the amount of solar radiation it receives.

The luminosity of the Sun (the total power produced) is

$$L_s = 4 \pi R_s^2 \sigma T_s^4$$

where R_s is the radius of the Sun and T_s its surface temperature, 5780 K. This is the amount of energy radiated each second by the Sun in all directions.

The Earth is a globe of radius R_E = 6370 km located an average distance $r_E = 1.50 \times 10^8$ km from the Sun. The amount of radiation emitted by the Sun that the Earth intercepts is determined by the ratio of the cross-sectional area of the Earth to the total area to which the radiation emitted by the Sun has spread at the distance of Earth (that is, the surface area of a sphere with a radius of Earth's distance from the Sun.) The amount of energy per second the Earth receives is therefore

$$P_E = L_s (\pi R_E^2 / 4\pi r_E^2).$$

The Earth absorbs this energy and then re-radiates it at longer wavelengths. The luminosity of the Earth is

$$L_E = 4 \pi R_E^2 \sigma T_E^4$$

according to the Stefan-Boltzmann law, where T_E is the average surface temperature of the Earth. If we ssume that the Earth absorbs all the energy that it receives and that it is also a steady-state system ($P_E = L_E$), it emits as much radiation as it receives.

Using the last three equations from Page 1, we can solve for the Earth's surface temperature:

$$T_E = (R_S / 2r_E)^{1/2} T_S$$

That is, the ratio of the Earth's surface temperature to that of the Sun depends only on the solar radius and the Earth-Sun distance.

Now, calculate the surface temperatures for the inner planets in the Solar System (in degrees Kelvin). Find out the distances to the planets from a reference book or online (for example, you can use the charts given in reference (*) below) and fill out the following table using the formulae above:

Planet	Mercury	Venus	Earth	Mars
Distance from the Sun (km)				
Calculated average surface temperature (K)				
Real average surface temperature* (K)	440	737	288	210

* Real average surface temperatures from NASA National Space Science Data Center's Planetary System Fact Sheets, http://nssdc.gsfc.nasa.gov/planetary/planetfact.html

Useful Formulae and Constants

Astronomical Unit (1 AU = average Earth-Sun distance) = 1.50×10^{11} m

Surface temperature of the Sun = 5780 K

Radius of the Sun = 696,000 km

Surface Area of Sphere= $4\pi r^2$ (where r is the radius of the sphere)

Questions:

1. Which planets have their calculated surface temperatures close to the real values? Why do you think this is the case?

2. Which planets have calculated surface temperatures that are way off? Why do you think this is the case? (Hint: Think about the assumptions made in the calculation.)

3. A habitable zone is the range of distances from the Sun (or from the star if we are talking about another star) where liquid water could exist. This is important because life as we know it is thought to be able to survive only in places where there is liquid water. What is the range of distances of the habitable zone around the Sun? (Hint: use the equations in Pages 1-2 to solve for the distance from the Sun at which the calculated average surface temperature is equal to freezing (273 K) or boiling (373 K).) How is Earth situated with regards to the habitable zone?

4. The calculations in this Worksheet assume that the amount of solar radiation is the only source influencing the surface temperature on a planet. Can you think of other sources that might influence the temperature? The calculations also assume that all of the solar radiation received by the planets is later re-radiated. Can you think of circumstances when this might not be the case?

5. During the MESSENGER mission to Mercury, the spacecraft will be exposed to temperatures much higher than those on Earth. You have calculated the difference in the surface temperatures of the two planets already. How much difference is there between the amount of energy experienced by objects at Mercury's distance compared with the situation at Earth's location? [Hint: calculate over how large an area the power of sunlight has spread by the time it reaches Mercury or Earth; what is the ratio?] Can you think of some ways to reduce the amount of heat experienced by the spacecraft?